CHEAT SHEETS HERO

Discrete Mathematics Cheatsheet

A comprehensive cheat sheet covering key concepts in Discrete Mathematics, including set theory, logic, relations, graph theory, and combinatorics. This serves as a quick reference for definitions, formulas, and common problem-solving techniques.

Set Theory

Basic Definitions

Set	A well-defined collection of distinct objects, considered as an object in its own right.
Element	An object in a set. Denoted by $\in$ (e.g., $x \in A$ means x is an element of set A).
Subset	A set A is a subset of B ($A \subseteq B$) if every element of A is also in B.
Proper Subset	A set A is a proper subset of B ($A \subset B$) if $A \subseteq B$ and $A \neq B$.
Universal Set (U)	The set containing all elements under consideration.
Empty Set (∅)	The set containing no elements. Also denoted by {}.

Set Operations

Union (∪)	$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
Intersection (∩)	$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
Difference (-)	$A - B = \{x \mid x \in A \text{ and } x \notin B\}$
Complement (A')	$A' = \{x \mid x \in U \text{ and } x \notin A\}$
Symmetric Difference (⊕)	$A \oplus B = (A - B) \cup (B - A)$
Cartesian Product (×)	$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

Set Identities

Identity Laws:
$A \cup \varnothing = A$
$A \cap U = A$

Domination Laws:
$A \cup U = U$
$A \cap \varnothing = \varnothing$

Idempotent Laws:
$A \cup A = A$
$A \cap A = A$

Complementation Law:
$(A')' = A$

Commutative Laws:
$A \cup B = B \cup A$
$A \cap B = B \cap A$

Associative Laws:
$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

Logic

Propositional Logic

Proposition	A declarative statement that is either true or false, but not both.
Conjunction (∧)	$p \wedge q$ is true if both p and q are true; otherwise, it is false.
Disjunction (∨)	$p \vee q$ is true if either p or q (or both) are true; it is false only if both are false.
Negation (¬)	$\neg p$ is true if p is false, and false if p is true.
Implication (→)	$p \rightarrow q$ is false only when p is true and q is false; otherwise, it is true. Also called a conditional statement.
Biconditional (↔)	$p \leftrightarrow q$ is true if p and q have the same truth value (both true or both false).

Logical Equivalences

De Morgan's Laws:
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Distributive Laws:
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Implication Equivalence:
$p \rightarrow q \equiv \neg p \vee q$

Biconditional Equivalence:
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Commutative Laws:
$p \wedge q \equiv q \wedge p$
$p \vee q \equiv q \vee p$

Associative Laws:
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Predicate Logic

Predicate	A statement involving variables. P(x) is the predicate P at x.
Quantifiers	Symbols that express the extent to which a predicate is true over a range of elements.
Universal Quantifier (∀)	$\forall x\, P(x)$ means P(x) is true for all x in the domain.
Existential Quantifier (∃)	$\exists x\, P(x)$ means there exists at least one x in the domain for which P(x) is true.
Negation of Quantifiers	$\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$ $\neg(\exists x\, P(x)) \equiv \forall x\, \neg P(x)$

Relations

Basic Definitions

Relation	A subset of $A \times B$, where A and B are sets. Represents a relationship between elements of A and B.
Binary Relation	A relation from a set A to itself (a subset of $A \times A$).
Reflexive Relation	A relation R on A is reflexive if $(a, a) \in R$ for all $a \in A$.
Symmetric Relation	A relation R on A is symmetric if $(a, b) \in R$ implies $(b, a) \in R$.
Transitive Relation	A relation R on A is transitive if $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$.
Equivalence Relation	A relation that is reflexive, symmetric, and transitive.

Properties of Relations

Irreflexive: A relation R on A is irreflexive if $(a, a) \notin R$ for all $a \in A$.

Antisymmetric: A relation R on A is antisymmetric if $(a, b) \in R$ and $(b, a) \in R$ implies $a = b$.

Partial Order: A relation that is reflexive, antisymmetric, and transitive.

Total Order: A partial order where for every $a, b \in A$, either $(a, b) \in R$ or $(b, a) \in R$.

Closures of Relations

Reflexive Closure	The smallest reflexive relation containing R. Add (a, a) for all a not already in the relation.
Symmetric Closure	The smallest symmetric relation containing R. If $(a, b) \in R$, add (b, a) to the relation.
Transitive Closure	The smallest transitive relation containing R. Computed using Warshall's Algorithm.

Graph Theory

Basic Definitions

Graph	A pair G = (V, E) where V is a set of vertices and E is a set of edges connecting these vertices.
Directed Graph (Digraph)	A graph where edges have a direction. Edges are ordered pairs (u, v).
Undirected Graph	A graph where edges have no direction. Edges are unordered pairs {u, v}.
Adjacent Vertices	Two vertices are adjacent if they are connected by an edge.
Degree of a Vertex	The number of edges incident to the vertex. In digraphs, indegree is the number of incoming edges, and outdegree is the number of outgoing edges.
Path	A sequence of vertices connected by edges.

Graph Properties

Connected Graph: A graph where there is a path between every pair of vertices.

Complete Graph (Kn): A graph where every pair of distinct vertices is connected by an edge.

Cycle: A path that starts and ends at the same vertex.

Tree: A connected graph with no cycles.

Bipartite Graph: A graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V.

Planar Graph: A graph that can be drawn in the plane without any edges crossing.

Graph Representations

Adjacency Matrix	A matrix representing the graph's connections. A[i][j] = 1 if there is an edge from vertex i to vertex j, and 0 otherwise.
Adjacency List	A list of adjacent vertices for each vertex in the graph.