Calculus Cheatsheet

A comprehensive calculus cheat sheet covering essential concepts, formulas, and techniques. This cheat sheet is designed to serve as a quick reference guide for students and professionals alike, providing a concise overview of calculus principles and methods.

Limits and Continuity

Limit Definitions

Formal Definition:	For every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < \lvert x - a \rvert < \delta$, then $\lvert f(x) - L \rvert < \varepsilon$.
Intuitive Definition:	As x approaches a, f(x) approaches L.
One-Sided Limits:	$\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$

Limit Laws

$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$
$\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, if $\lim_{x \to a} g(x) \neq 0$

Continuity

Definition:	A function f(x) is continuous at x = a if $\lim_{x \to a} f(x) = f(a)$. This means that f(a) exists, the limit exists, and they are equal.
Types of Discontinuities:	Removable, Jump, Infinite

Derivatives

Basic Differentiation Rules

Power Rule:	$\frac{d}{dx}(x^n) = nx^{n-1}$
Constant Rule:	$\frac{d}{dx}(c) = 0$
Constant Multiple Rule:	$\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x))$
Sum/Difference Rule:	$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

Product and Quotient Rules

Product Rule:	$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
Quotient Rule:	$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule

Chain Rule:	$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

Derivatives of Trig Functions

$\frac{d}{dx}(\sin x)$	$\cos x$
$\frac{d}{dx}(\cos x)$	$-\sin x$
$\frac{d}{dx}(\tan x)$	$\sec^2 x$
$\frac{d}{dx}(\csc x)$	$-\csc x \cot x$
$\frac{d}{dx}(\sec x)$	$\sec x \tan x$
$\frac{d}{dx}(\cot x)$	$-\csc^2 x$

Integrals

Basic Integration Rules

Power Rule:	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$
Constant Rule:	$\int c\, dx = cx + C$
Constant Multiple Rule:	$\int cf(x)\, dx = c \int f(x)\, dx$
Sum/Difference Rule:	$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$
$\int \frac{1}{x} dx$	$\ln\lvert x\rvert + C$
$\int e^x dx$	$e^x + C$

Integration by Parts

Formula:	$\int u\, dv = uv - \int v\, du$

Trigonometric Integrals

$\int \sin x\, dx$	$-\cos x + C$
$\int \cos x\, dx$	$\sin x + C$
$\int \sec^2 x\, dx$	$\tan x + C$
$\int \csc^2 x\, dx$	$-\cot x + C$
$\int \sec x \tan x\, dx$	$\sec x + C$
$\int \csc x \cot x\, dx$	$-\csc x + C$

Trigonometric Substitution

Use when you have integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, or $\sqrt{x^2 - a^2}$. Substitute $x = a\sin \theta$, $x = a\tan \theta$, or $x = a\sec \theta$ respectively.

Applications of Derivatives

Related Rates

Identify the variables, find the equation relating them, differentiate with respect to time, and solve for the desired rate.

Optimization

Find critical points by setting the first derivative to zero or undefined, then use the first or second derivative test to determine local maxima and minima. Check endpoints for absolute extrema.

L'Hôpital's Rule

When to Use:	For limits of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.
Rule:	$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Mean Value Theorem

Theorem:	If f is continuous on [a, b] and differentiable on (a, b), then there exists a c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$