Equations

A concise reference for fundamental equations and formulas across various mathematical and scientific domains. From basic algebra to advanced calculus and physics, this cheat sheet provides a quick guide to key concepts and their applications.

Algebraic Formulas

Basic Identities

Square of a Sum:
(a + b)$^{2 = a}$2 + 2ab + b^2
Square of a Difference:
(a - b)$^{2 = a}$2 - 2ab + b^2
Difference of Squares:
a$^{2 - b}$2 = (a + b)(a - b)
Cube of a Sum:
(a + b)$^{3 = a}$3 + 3a$^{2b + 3ab}$2 + b^3
Cube of a Difference:
(a - b)$^{3 = a}$3 - 3a$^{2b + 3ab}$2 - b^3
Sum of Cubes:
a$^{3 + b}$3 = (a + b)(a$^{2 - ab + b}$2)
Difference of Cubes:
a$^{3 - b}$3 = (a - b)(a$^{2 + ab + b}$2)

Quadratic Formula

For a quadratic equation of the form $ax^2 + bx + c = 0$, the solutions for x are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant:

$$\Delta = b^2 - 4ac$$

- If $\Delta > 0$, there are two distinct real roots.
- If $\Delta = 0$, there is exactly one real root.
- If $\Delta < 0$, there are no real roots (two complex conjugate roots).

Logarithms and Exponents

Product Rule:	$\log_b(mn) = \log_b(m) + \log_b(n)$
Quotient Rule:	$\log_b(\frac{m}{n}) = \log_b(m) - \log_b(n)$
Power Rule:	$\log_b(m^p) = p \log_b(m)$
Change of Base:	$\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$
Exponential Growth:	$A = P(1 + r)^t$
Exponential Decay:	$A = P(1 - r)^t$

Geometric Formulas

Area Formulas

Square:	$A = s^2$
Rectangle:	$A = lw$
Triangle:	$A = \frac{1}{2}bh$
Circle:	$A = \pi r^2$
Trapezoid:	$A = \frac{1}{2}(b_1 + b_2)h$
Parallelogram:	$A = bh$

Volume Formulas

Cube:	$V = s^3$
Rectangular Prism:	$V = lwh$
Sphere:	$V = \frac{4}{3}\pi r^3$
Cylinder:	$V = \pi r^2 h$
Cone:	$V = \frac{1}{3}\pi r^2 h$
Pyramid:	$V = \frac{1}{3}Bh$

Surface Area Formulas

Cube:	$SA = 6s^2$
Rectangular Prism:	$SA = 2(lw + lh + wh)$
Sphere:	$SA = 4\pi r^2$
Cylinder:	$SA = 2\pi r^2 + 2\pi rh$
Cone:	$SA = \pi r^2 + \pi rl$

Trigonometric Formulas

Basic Trigonometric Functions

Sine:	$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$
Cosine:	$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$
Tangent:	$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
Cosecant:	$\csc(\theta) = \frac{1}{\sin(\theta)}$
Secant:	$\sec(\theta) = \frac{1}{\cos(\theta)}$
Cotangent:	$\cot(\theta) = \frac{1}{\tan(\theta)}$

Pythagorean Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$
$1 + \tan^2(\theta) = \sec^2(\theta)$
$1 + \cot^2(\theta) = \csc^2(\theta)$

Angle Sum and Difference Formulas

Sine Sum:	$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin($
Sine Difference:	$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin($
Cosine Sum:	$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin($
Cosine Difference:	$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin($
Tangent Sum:	$\tan(A + B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$
Tangent Difference:	$\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$

Calculus Formulas

Basic Derivatives

Power Rule:	$\frac{d}{dx}(x^n) = nx^{n-1}$
Constant Multiple Rule:	$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$
Sum Rule:	$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$
Product Rule:	$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$
Quotient Rule:	$\frac{d}{dx}(\frac{f(x)}{g(x)}) = \frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$
Chain Rule:	$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$

Basic Integrals

Power Rule:	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
Integral of 1/x:	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
Integral of e^x:	$\int e^x dx = e^x + C$
Integral of sin(x):	$\int \sin(x) dx = -\cos(x) + C$
Integral of cos(x):	$\int \cos(x) dx = \sin(x) + C$

Fundamental Theorem of Calculus

Part 1: If f is continuous on $[a, b]$, then the function F defined by

$$F(x) = \int_a^x f(t)dt \quad a \leq x \leq b$$

is an antiderivative of f, that is, $F'(x) = f(x)$.

Part 2: If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, $F' = f$.

Equations Cheat Sheet

Algebraic Equations

Linear Equation:
$ax + b = 0$
Solve for x:
$x = -\frac{b}{a}$

Quadratic Equation:
$ax^2 + bx + c = 0$
Quadratic Formula:
$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

System of Linear Equations (2 variables):
$a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$
Solve using substitution, elimination, or matrices.

Factoring:
$a^2 - b^2 = (a+b)(a-b)$
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

Polynomial Equations:
Use factoring, synthetic division, or numerical methods to find roots.

Exponential Equations:
$a^x = b$
Solve using logarithms:
$x = \frac{\log(b)}{\log(a)}$

Logarithmic Equations:
$\log_a(x) = b$
Convert to exponential form:
$x = a^b$

Absolute Value Equations:
$\lvert x\rvert = a$
Then:
$x = a \text{ or } x = -a$

Rational Equations:
Clear denominators by multiplying all terms by the least common denominator.

Calculus Equations

Derivative Rules:
Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$
Constant Rule: $\frac{d}{dx}c = 0$
Product Rule: $\frac{d}{dx}(uv) = u'v + uv'$
Quotient Rule: $\frac{d}{dx}(\frac{u}{v}) = \frac{u'v-uv'}{v^2}$
Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Integral Rules:
Power Rule: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
Exponential: $\int e^x dx = e^x + C$
Trigonometric: $\int \sin(x)dx = -\cos(x) + C$
$\int \cos(x)dx = \sin(x) + C$

Fundamental Theorem of Calculus:
$\int_a^b f(x)dx = F(b) - F(a)$
where $F'(x) = f(x)$

Limits:
$\lim_{x\to a} f(x)$
Evaluate by direct substitution, factoring, rationalizing, or L'Hôpital's Rule.

L'Hôpital's Rule:
If $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$
then
$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

Taylor Series:
$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

Integration by Parts:
$\int u dv = uv - \int v du$

Partial Derivatives:
$\frac{\partial f}{\partial x}$
Derivative of f with respect to x while holding other variables constant.

Gradient:
$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

Trigonometric Equations

Basic Identities:
$\sin^2(\theta) + \cos^2(\theta) = 1$
$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Pythagorean Identities:
$1 + \tan^2(\theta) = \sec^2(\theta)$
$1 + \cot^2(\theta) = \csc^2(\theta)$

Double Angle Formulas:
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$

Sum and Difference Formulas:
$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$
$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$
$\tan(a \pm b) = \frac{\tan(a)\pm\tan(b)}{1\mp\tan(a)\tan(b)}$

Solving Trigonometric Equations:
Isolate the trigonometric function, then use inverse trigonometric functions to find solutions. Consider the period and general solutions.

Law of Sines:
$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Law of Cosines:
$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = a^2 + b^2 - 2ab\cos(C)$

Half Angle Formulas:
$\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
$\cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$
$\tan(\frac{\theta}{2}) = \frac{1-\cos(\theta)}{\sin(\theta)}$

Product-to-Sum Formulas:
$\sin(a)\cos(b) = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$
$\cos(a)\sin(b) = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$
$\cos(a)\cos(b) = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$
$\sin(a)\sin(b) = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$

Physics Equations

Kinematics:
$v = v_0 + at$
$x = x_0 + v_0 t + \frac{1}{2}at^2$
$v^2 = v_0^2 + 2a(x - x_0)$

Newton's Second Law:
$F = ma$

Work and Energy:
$W = Fd\cos(\theta)$
$KE = \frac{1}{2}mv^2$
$PE = mgh$

Momentum:
$p = mv$

Impulse:
$J = \Delta p = F\Delta t$

Gravitational Force:
$F = G\frac{m_1 m_2}{r^2}$

Ohm's Law:
$V = IR$

Power (Electrical):
$P = IV = I^2R = \frac{V^2}{R}$

Wave Equation:
$v = f\lambda$

Ideal Gas Law:
$PV = nRT$

Geometric Equations

Area of a Circle:
$A = \pi r^2$

Circumference of a Circle:
$C = 2\pi r$

Area of a Triangle:
$A = \frac{1}{2}bh$

Pythagorean Theorem:
$a^2 + b^2 = c^2$

Volume of a Sphere:
$V = \frac{4}{3}\pi r^3$

Surface Area of a Sphere:
$A = 4\pi r^2$

Volume of a Cylinder:
$V = \pi r^2 h$

Surface Area of a Cylinder:
$A = 2\pi rh + 2\pi r^2$

Area of a Rectangle:
$A = lw$

Perimeter of a Rectangle:
$P = 2l + 2w$

Financial Equations

Simple Interest:
$I = PRT$
Where:
- I = Interest
- P = Principal
- R = Rate
- T = Time

Compound Interest:
$A = P(1 + \frac{r}{n})^{nt}$
Where:
- A = Amount
- P = Principal
- r = interest rate
- n = number of times interest applied per time period
- t = number of time periods elapsed

Present Value:
$PV = \frac{FV}{(1+r)^n}$
Where:
- PV = Present Value
- FV = Future Value
- r = Discount rate
- n = Number of periods

Future Value:
$FV = PV(1 + r)^n$
Where:
- FV = Future Value
- PV = Present Value
- r = Interest rate
- n = Number of periods

Annuity (Future Value):
$FV = P \times \frac{((1+r)^n - 1)}{r}$
Where:
- FV = Future Value
- P = Periodic Payment
- r = Interest Rate
- n = Number of periods

Annuity (Present Value):
$PV = P \times \frac{(1-(1+r)^{-n})}{r}$
Where:
- PV = Present Value
- P = Periodic Payment
- r = Interest Rate
- n = Number of periods

Mortgage Payment:
$M = P\frac{r(1+r)^n}{(1+r)^n - 1}$
Where:
- M = Monthly Payment
- P = Principal Loan Amount
- r = Monthly Interest Rate
- n = Number of Months

Return on Investment (ROI):
$ROI = \frac{NetProfit}{CostofInvestment} \times 100$

Statistics Equations

Mean:
$\mu = \frac{\sum_{i=1}^{n} x_i}{n}$

Variance:
$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$

Standard Deviation:
$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$

Z-Score:
$z = \frac{x - \mu}{\sigma}$

Correlation Coefficient:
$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$

Regression Equation:
$y = a + bx$
where
$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$
and
$a = \bar{y} - b\bar{x}$

Probability:
$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$

Conditional Probability:
$P(A|B) = \frac{P(A \cap B)}{P(B)}$

Bayes' Theorem:
$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$